

Some Problems of Electric Spectrometry of Aerosol Particles

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1. Introduction

A principle: If we wish to know numerical values of n independent scalar parameters then we should do at least n scalar measurements.

First conclusion: If our instrument issues n scalar measurements then we cannot find more than n values or scalar parameters of the size distribution.

Second conclusion: Before starting to develop a method of measurement, we should choose a model of size distribution, where the distribution is uniquely determined by a small number n of numerical values.

2. Models of size distribution

Continuous model: most fundamental in theory, but it has infinite number of degrees of freedom.

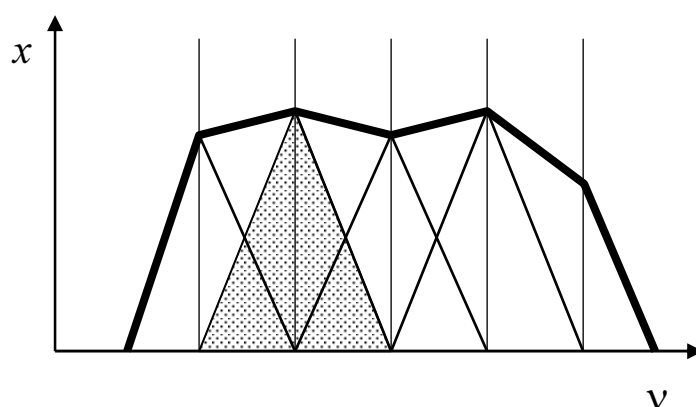
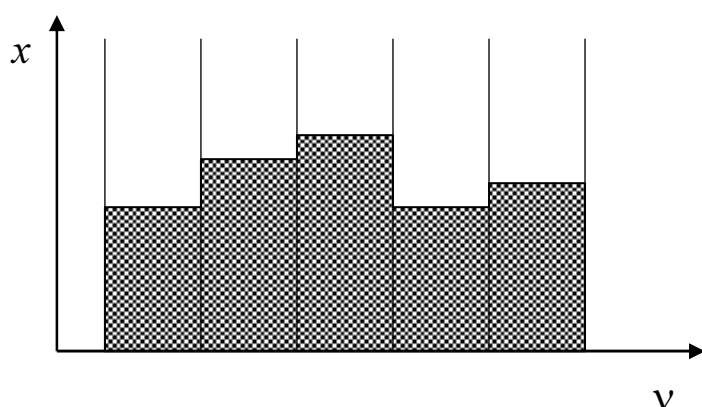
Fraction model: finite number of degrees of freedom, but there is a problem how to associate the set of fraction concentrations with the continuous model.

Finite-dimensional linear model:

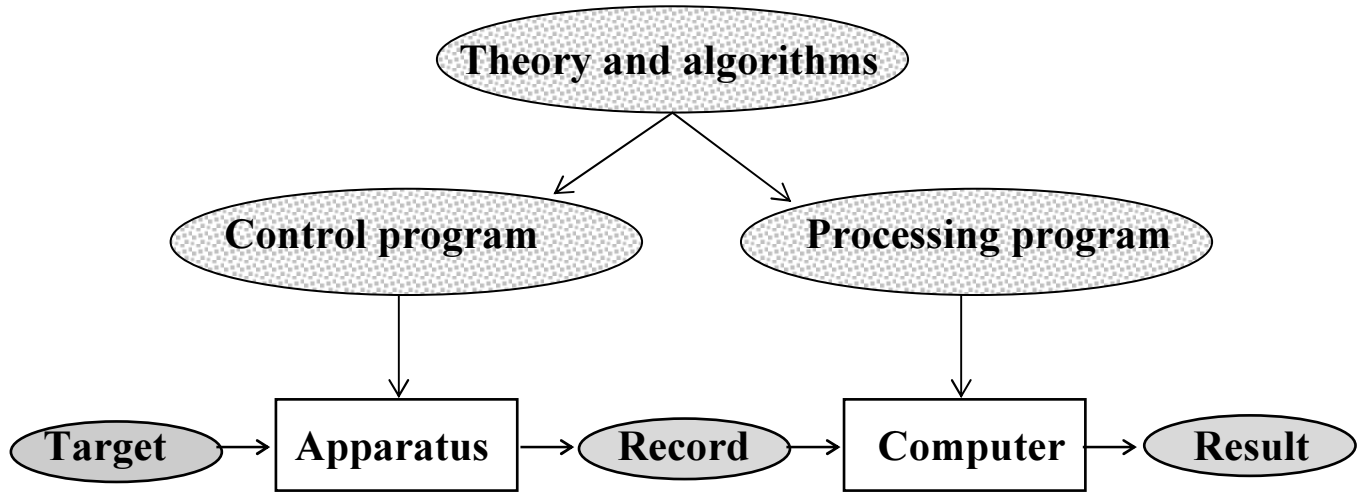
Argument (size) v , density of distribution x . Basis $\{f^i(v)\}$, $i = 1..n$.

$$x(v) = \sum_{i=1}^n x_i f^i(v)$$

Examples: A simple fraction model and a piecewise linear model



2. Mathematical model of a linear spectrometer



Target: Aerosol

Record: The set of channel signals $\mathbf{y} = \{y_1, y_1, \dots, y_m\}$, m is number of channels.

Result: The asked distribution presented by n numbers $\mathbf{x} = \{x_1, x_1, \dots, x_n\}$

Equation of the linear spectrometer:

$$y_i = \sum_{j=1}^n g_{ij} x_j + \xi_i, \quad i = 1 \dots m, \quad \xi \text{ is measuring noise}$$

$$\mathbf{y} = \{y_i\}, \quad \mathbf{G} = \{g_{ij}\}, \quad \mathbf{x} = \{x_j\}, \quad \boldsymbol{\xi} = \{\xi_j\},$$

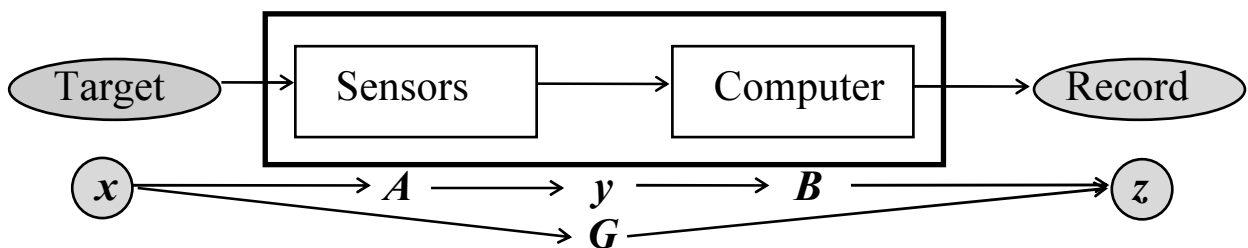
$$\mathbf{y} = \mathbf{G}\mathbf{x} + \boldsymbol{\xi}$$

Applications:

Apparatus record \rightarrow mobility distribution

Mobility distribution \rightarrow size distribution

Apparatus record \rightarrow size distribution



$$\mathbf{y} = \mathbf{A}\mathbf{x} \quad \mathbf{z} = \mathbf{B}\mathbf{y} \quad \mathbf{z} = \mathbf{B}(\mathbf{A}\mathbf{x}) = (\mathbf{B}\mathbf{A})\mathbf{x}$$

Conclusion: $\mathbf{z} = \mathbf{G}\mathbf{x}$, where $\mathbf{G} = \mathbf{B}\mathbf{A}$

Example: Apparatus matrix of EAS according to Aadu Mirme

[illegible]

3. Data processing

Traditionally, the infinite dimensional model is used in the theory. The infinite dimensional apparatus equation

$$y(w) = \int_{v_{\min}}^{v_{\max}} g(w, v) x(v) dv$$

is a Fredholm equation of the first kind. It is incorrectly set up and cannot be solved without additional information.

Our equation is an algebraic linear equation

$$\mathbf{y} = \mathbf{G}\mathbf{x} + \boldsymbol{\xi} \quad \text{or} \quad y_j = \sum_{i=1}^n G_{ji} x_i + \xi_j$$

It can be solved using Gauss-Markoff least squares algorithm:

$$\hat{\mathbf{x}} = \mathbf{A}^{-1} \mathbf{b} = (\mathbf{G}^T \mathbf{D}^{-1} \mathbf{G})^{-1} \mathbf{G}^T \mathbf{D}^{-1} \mathbf{y}$$

The algorithm:

$$\begin{aligned} \mathbf{H} &:= \mathbf{G}^T \mathbf{D}^{-1} \\ \mathbf{C} &:= (\mathbf{H} \mathbf{G})^{-1} \\ \mathbf{x} &:= \mathbf{C} \mathbf{H} \mathbf{y} \end{aligned}$$

\mathbf{D} is the covariance (or dispersion) matrix of record noise. Its diagonal elements are the variances σ^2 and non-diagonal elements are covariances $\rho_{i\varphi} \sigma_i \sigma_j$. \mathbf{C} is the covariance matrix of the spectrum and the estimates of the measuring errors can be found as its diagonal elements.

Example: Let $G_{ji} = c \exp(-c^2(j-i)^2)$

Then less the coefficient c then better is the resolution. What happens if we will reduce the value of c ? Let us calculate the coefficient of noise amplification as the square root of the ratio of measurement errors in the result and in the record:

